Facit til kursusgang 24: Ubestemte integraler 3

1. Svarene er:

$$-\frac{1}{3}\cos(3x+1)+c, \qquad -\frac{1}{2}\sin(-2x+1)+c, \qquad \frac{1}{5}e^{5x-3}+c.$$

2. Svarene er

$$\ln(x^3+x-1)+c, \qquad \frac{1}{6}(x+1)^6+c, \qquad \frac{1}{4}(\frac{1}{2}x^4-x+12)^4+c.$$

3. Svarene er

$$\frac{\ln(x^2+1)}{2}+c, \qquad \frac{e^{x^3-1}}{3}+c, \qquad \frac{\sin(x^2-1)}{2}+c.$$

4. Svarene er:

$$\frac{1}{2}e^{x^2}+c, \qquad (x+1)\ln(x+1)-(x+1)+c, \qquad -\cos(x^2-1)+c.$$

5. Svarene er:

$$\int \cos(x)\sin(x)\,dx = \frac{1}{2}\int \sin(2x)\,dx = -\frac{1}{4}\cos(2x)+c.$$
$$\int \sin^3(x)\cos(x)\,dx = \frac{1}{4}\sin^4(x)+c.$$
$$\int (3x^2-1)\cos(x^3-x+2)\,dx = \sin(x^3-x+2)+c.$$

6. Svarene er:

$$-\frac{1}{a}\cos(ax+b)+c \qquad \frac{1}{a}\sin(ax+b)+c$$

$$\frac{1}{a}e^{ax+b}+c \qquad \frac{1}{a}((ax+b)\ln(ax+b)-(ax+b))+c$$

7. Svaret er:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2e^{\sqrt{x}}+c.$$

EKSTRAOPGAVER:

8. Da $\tan x = \frac{\sin x}{\cos x}$ kan vi substituere $u = \cos(x)$ og få at $\frac{du}{-\sin(x)} = dx$, hvilket giver

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\int \frac{1}{u} \, du = -\ln(u) + c = -\ln(\cos(x)) + c.$$

9. Vi har at

$$\frac{d}{dx}(F(g(h(x))) + c) = F'(g(h(x)))g'(h(x))h'(x) = f(g(h(x))g'(h(x))h'(x).$$

10. Lad os først bruge integration ved substitution til at udregne

$$\int (x+1)^a \, dx,$$

for $a \neq -1$. Substituerer vi $u = x + 1$ får vi at $du = dx$ og

$$\int (x+1)^a \, dx \int u^a \, du = \frac{1}{a+1} u^{a+1} = \frac{1}{a+1}(x+1)^{a+1}.$$

Ved at bruge denne mellemregning for $a = \frac{1}{2}$ og $a = \frac{3}{2}$ får vi at

$$\begin{aligned} \int x\sqrt{x+1} \, dx &= \frac{2}{3} x(x+1)^{\frac{3}{2}} - \frac{2}{3} \int (x+1)^{\frac{3}{2}} \, dx \\ &= \frac{2}{3} x(x+1)^{\frac{3}{2}} - \frac{2}{3}\frac{2}{5}(x+1)^{\frac{5}{2}} + c \\ &= \frac{2}{3} x(1+x)^{\frac{3}{2}} - \frac{4}{15}(1+x)^{\frac{5}{2}} + c. \end{aligned}$$

For at udregne

$$\int \sqrt{1+\sqrt{x}} \, dx$$

substituerer vi $u = \sqrt{x}$ og får

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad \Leftrightarrow \quad 2\sqrt{x} du = dx \quad \Leftrightarrow \quad 2u du = dx.$$

Dette giver at

$$\begin{aligned} \int \sqrt{1+\sqrt{x}} \, dx &= 2 \int u\sqrt{1+u} \, du \\ &= \frac{4}{3} u(1+u)^{\frac{3}{2}} - \frac{8}{15}(1+u)^{\frac{5}{2}} + c \\ &= \frac{4}{3}\sqrt{x}(1+\sqrt{x})^{\frac{3}{2}} - \frac{8}{15}(1+\sqrt{x})^{\frac{5}{2}} + c. \end{aligned}$$

Bemærk at begge facit kan reduceres yderligere.