Plastic structure with vibration reducing means

D. Moradi, I. Pîrlea, M. Mugica, S. Gutiérrez, V. Manurkar

Department of Materials and Production, Aalborg University
Fibigerstraede 16, DK-9220 Aalborg East, Denmark
Email: 4105af18@student.aau.dk,
Web page: http://www.mechman.m-tech.aau.dk/

Abstract

This paper presents an analysis of the effect of geometry variations in the transmission of vibrations through a plastic structure. The structure in focus is a simplified model of an industrial vacuum cleaner. Its shape is cylindrical, with top and bottom plates closing it, where there is a motor attached to the bottom plate. The vibrations produced by the motor are transmitted to the top plate, where they are radiated as noise. Hence, this paper is focused on developing a method for reducing the noise levels by optimizing the design of the bottom plate. An experimental test structure is used to analyze the response under working conditions. These results are used to validate the FE analysis conducted for the same test structure. Taking this model as a basis, an optimization procedure is performed by changing the geometry in terms of shape and thickness. From this, an optimal design is found which reduces the emissions of noise at a particular frequency. Lastly, the new design is manufactured to test if the proposed optimization strategy provides the expected improvement.

Keywords: Vibrations, Structural acoustics, Optimization, Resonance, Mode shapes, Damping, Harmonic response

1. Introduction

The vacuum cleaner manufacturing company Nilfisk, has proposed a project regarding reduction of noise in their products. Nilfisk is specialized in producing high speed vacuum cleaners for industrial use, with a motor speed of 30,000 rpm or above. At these speeds, any imbalance in the force produces vibrations which propagate through the structure and are radiated by the surface of the vacuum cleaner as audible noise.

The structure under study provided by Nilfisk is a simplified model of the vacuum cleaner. According to the initial guidelines, the geometry consists of a cylinder sealed with plates, both on the top and on the bottom (see Fig. 1). The motor constitutes a source of vibrations at the centre of the bottom plate, which are then transmitted through the whole structure.

As the structure vibrates, it radiates noise to the surrounding air. The top plate is the main source of sound emission, and thus, the measurements and analysis are focused on its surface. In addition to this, a few limitations regarding the redesign of the structure are provided: bottom plate is the only part which can be modified in terms of shape and thickness; minimum thickness is set to 1 mm and maximum to 6 mm; holes are discarded as the structure has to remain sealed.

Changing the geometry of the plate, non-uniformities are created in order to impede the transmission of vibration energy created by the source. By doing this, the natural frequencies of the system are shifted, leading to a possible reduction of the amplitude response in the harmonic analysis.

Besides this, the normal operating speeds for the vacuum cleaner are in the range of 30,000-42,000 rpm which is equivalent to 500-700 Hz. Hence, the frequency range examined is restricted to this. In addition, Nilfisk is concerned about the response of top plate at 672 Hz, wherein they conduct their noise measurements for the quality tests.

To analyze this, a Finite Element (FE) model is created with the geometry of the simplified model in order to evaluate the modal and harmonic responses. Furthermore, several experiments are conducted with the purpose of validating the obtained results. This model serves as the basis for the optimization strategy to be developed, from which the new design is proposed.

Eventually, a prototype is manufactured, to test if it produces a sufficient reduction of noise and to evaluate the optimization strategy developed.

A quantity of the level of vibrations is to be compared in the different simulations and experiments through the project. Due to the limitations of the FE analysis and the measuring devices, acceleration and displacement are chosen to serve as the main measurements.

In addition to the article, an extended version of the conducted study and a more detailed description of the procedure and results obtained can be found in [1].

2. FE model

To begin with the process, a finite element model of the experimental setup is developed. The ultimate goal is to narrow the mode shapes of interest and have an insight of the behaviour of the original design. This model is based on the geometry and material characteristics provided by Nilfisk (see Fig. 1 and Tab. I).

The damping material serves to replicate the characteristics of the structure and to reduce the displacements of the top plate. On the other hand, the layers of foam are not considered (see Fig. 1) in this model, since the airborne sound is neglected in this analysis. Apart from that, it is assumed that the gluing between the cylinder and the plates is ideal, and hence, they are modelled as one piece.

Furthermore, for the boundary conditions of the model two cases are considered: one in which the lower edge of the bottom plate is fixed, and another one using soft springs instead. The latter one is more complicated, but is closer to the experimental setup, as discussed in Section 3.1. In contrast, fixing the displacements is easier to implement but may affect the results. However, after analyzing the response of the part it is concluded that the tendencies of both cases are similar. The amplitudes are slightly larger for the springs, as expected, due to rigid body motion.

Therefore, it is decided to use the fixed displacement boundary condition for the analyses to come in the paper.

2.1 Modal analysis

After the model is created, a modal analysis is performed in order to compute the natural frequencies and consequently, by inspecting the associated mode shapes, the critical frequencies are identified [3]. These frequencies correspond to the mode shapes that tend to emit the most noise. The modal behavior is analyzed on the frequency range of 0-700 Hz.

The symmetric mode shapes have a higher sound

Tab. I Material properties of the simplified model provided by Nilfisk [2]

Part	Density (kg/m^3)	Young's Modulus (MPa)	Poisson's ratio
Structure	1190	3210	0.39
Damping plate	1310	332	0.49

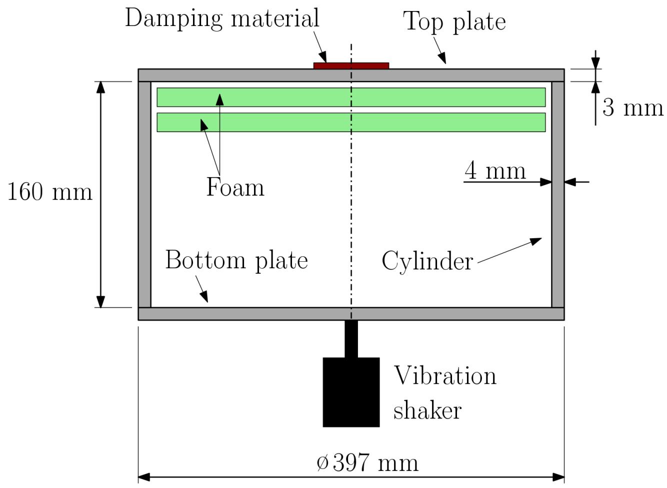

Fig. 1 Simplified model

radiation [4] [5], which, in the case of this model are the axisymmetric ones: at 546 Hz for the bottom and at 576 Hz for the top plate. An example of an axisymmetric mode is shown in Fig. 2. Taking this into account, they should be shifted away from normal operating conditions to avoid excessive vibrations.

2.2 Harmonic analysis

For this study, a periodic point load is applied in the center of the bottom plate. Then, the amplitude responses are recorded at the center of the top and bottom plates, as shown in Fig. 3. In this case, the range of frequencies is reduced to 450-700 Hz in order to put more emphasis on the problematic frequency range.

A peak appears as a consequence of the load applied exciting a resonant frequency, which corresponds to an axisymmetric mode of the bottom plate (see Fig.2). Since the vibration energy generated in the bottom plate is transmitted to the top plate, when resonance occurs on the bottom plate more energy is transferred to the top plate.

In conclusion, the resonant frequencies of the bottom plate have to be changed, such that the amplitude response of the top plate is lowered, and thus, reducing the noise emitted.

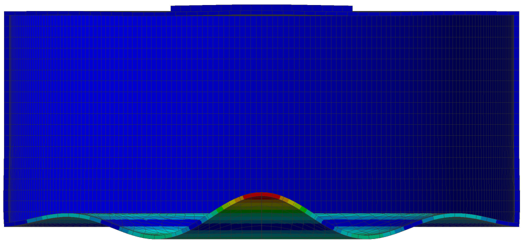

Fig. 2 Noisy mode at 546 Hz

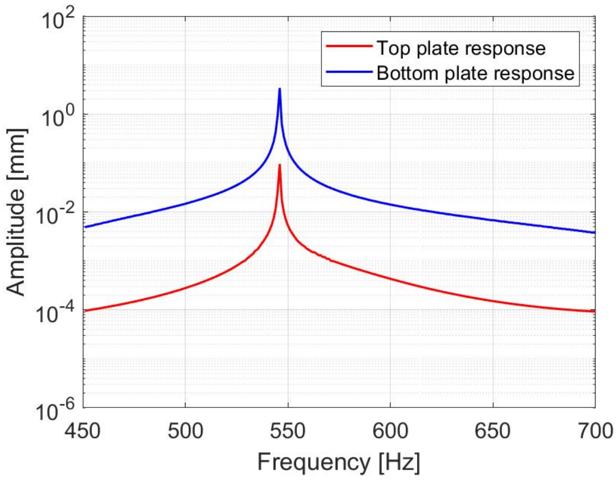

Fig. 3 Harmonic response

3. Experiments

To verify the FE model, experiments have been conducted on the model provided by Nilfisk. These experiments consist of different analyses regarding harmonic response and mode shapes. The program used to conduct the experiments is *PULSE Reflex*.

3.1 Experimental setup

The setup consists of the simplified model, the test structure and measuring devices. The measurement devices used are accelerometers, which measure unidirectional accelerations at particular points. The specimen is chosen to be freely supported to conduct the analyses, such that the same conditions can be simulated in the FE model. This is done by suspending the specimen with elastic bands of low stiffness tied to the test structure. A more detailed explanation of the setup is discussed in [1]. The final setup resembles the one shown in Fig. 4.

The accelerometers are positioned as such to capture the behaviour of the critical mode shapes. Taking this into account, three points on the top surface are chosen to place the accelerometers. The first position is the nearest to the damper plate, at a distance of 80 mm from the centre. Secondly, another accelerometer is placed at a distance of 127 mm from the centre where higher

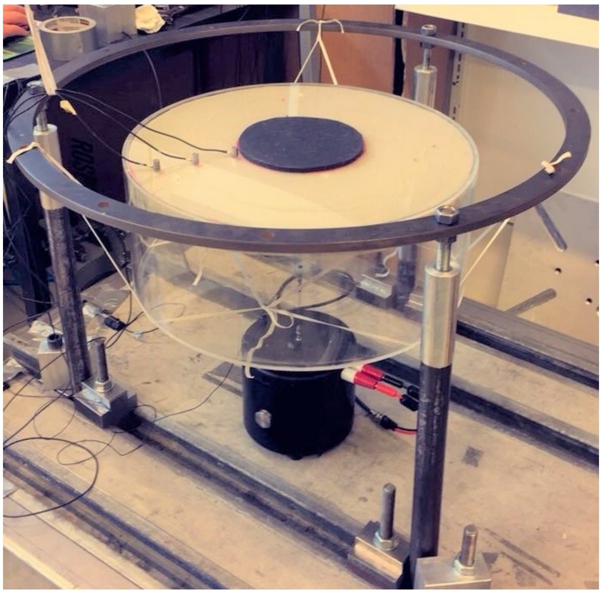

Fig. 4 Test setup

accelerations are expected. Finally, an accelerometer is placed at a distance of 173 mm away from the centre where the smallest accelerations are recorded in the FE model.

3.2 Results

For these experiments, the measurements are recorded from 0 Hz to 800 Hz. However, the shaker is not exciting the frequencies below 40 Hz, so the accelerations recorded outside this range is considered as noise.

Initially, a random noise test is prepared so that the structure is excited from 40 Hz to 800 Hz. Random noise is a way to excite the structure with a varying signal, that never repeats itself. The results from this recording are given as accelerations, in the form of the 'Autospectrum' in *PULSE Reflex*. These values must be post-processed, so that the frequency response is obtained. The final result is shown in Fig. 5. [6].

To verify the results acquired with the random noise test, a measurement using a periodic chirp is also done. The periodic chirp is a harmonic force that varies in frequency. The response from it corresponds to the results obtained from the random noise.

Furthermore, an experiment with a sinusoidal force exciting at a specific frequency is done to verify both results. The results show that the type of forcing does not have a significant effect on the response of the system. This is as expected, since the acceleration is normalized with respect to the force. Thereby, the

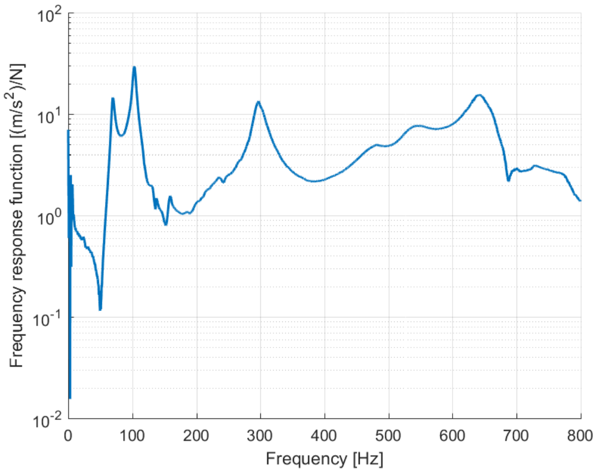

Fig. 5 Response of the accelerometer at 80 mm from PULSE Reflex

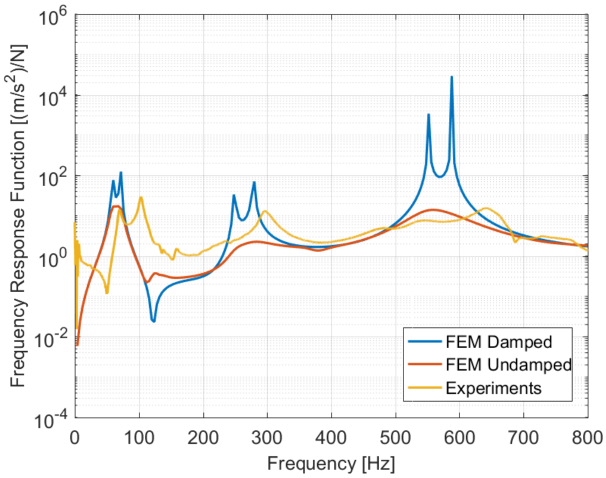

Fig. 6 Comparison of the responses

measurements from the random noise analysis are validated and used in the comparison. This investigation is further explained in [1].

Lastly, to validate the mode shapes from the FE model, a modal analysis is performed with *PULSE Reflex* on the experimental setup. The results are presented in [1], and show that the mode shapes coincides with the ones extracted from the FE model.

3.3 Uncertainty assessment

An investigation of the uncertainty is performed as a necessity to account for how much the measurements deviate. This is done with the use of the 'Coherence' measured in *PULSE Reflex* [6]. Once this is known, an uncertainty assessment is conducted [1], giving an estimated error of around 1 %.

3.4 Comparison

A comparison between the FE model and the experimental results is performed to validate it. This is significant since the FE model explained earlier, is the one which is going to be utilized for optimization procedures. For the sake of simplicity, only one accelerometer is analyzed for comparing the two methods, while the complete study can be found in [1].

Initially, the FE model does not include any damping. Nevertheless, as it can be observed in Fig.6, the response of the experiments seems to be damped when compared to the FE results. The simplified model may be influenced by several sources of damping, such as the structure itself, the damping plate and the gluing of the parts.

Taking this argument into account, a comparison is done by introducing a damping ratio of 0.287 for the damper plate and 0.06 for the structure.

The response shown in Fig.6 is satisfactory as the tendencies of both methods behave similarly. Therefore, it can be concluded that the FE model is suitable to proceed with for the optimization.

4. Optimization

Once the problematic resonant frequencies are identified, the objective of the optimization can be determined to improve the behaviour of the system. Knowing that the frequency range is established to 500-700 Hz, the cost function is set to move the frequency peak at 546 Hz (shown in Fig.3) out of the range. In addition to this, the amplitude response in the harmonic analysis is to be minimized, thus, reducing the emitted noise as much as possible.

To do so, the geometry of the bottom plate is modified in order to reduce the transmission of energy waves created by the vibration of the motor. This geometry is defined by several design variables which are varied through the optimization process [7]. In this way, the input model for the vibrations study in each iteration of the optimization procedure is updated as a function of the design variables.

To reduce the transmission of energy waves through the structure, two strategies are tried:

- Changing the thicknesses of the bottom plate to create non-uniformities in the geometry. To do so, a parametric model is created, such that the bottom

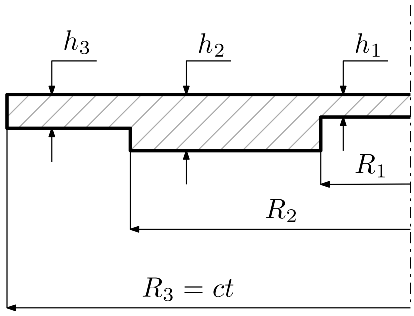

Fig. 7 Stepped geometry

plate is divided in three different volumes radially. These volumes are defined by five design variables: three thicknesses (h_1, h_2, h_3), inner and outer radii (R_1, R_2). In Fig. 7 these parameters are shown.

- Keeping the thickness constant while changing the shape of the cross-section. The curved shape is obtained making use of a spline defined by four key-points (K_1, K_2, K_3 and K_4). These key-points are located using the design variables (Z_1, Z_2, Z_3, R_1 and R_2), which define the shape of the curved cross-section, as shown in Fig. 8.

4.1 Stepped cross-section optimization

This strategy is divided in several stages. In the first stage, the thicknesses are varied while keeping the radii constant. So that a simpler model with three design variables is optimized making use of different cost function formulations. Each of them is evaluated with the Sequential Quadratic Programming (SQP) algorithm [8] and the best one is chosen for the latter strategies, which are explained in detail in [1]. The cost function formulations analyzed are:

- Minimize the maximum amplitude in the range of interest.
- Use a multi-objective formulation with the five biggest amplitude values. From this, a scalarized cost function is obtained which is then minimized.
- Add a cost function to identify the frequency peak using the Modal Assurance Criterion (MAC) [9]. With this, two cost functions are obtained: one to identify and reduce the resonant frequency of the bottom plate; and the other to minimize the amplitude of the response. Then, these two are merged together giving a bigger influence to the amplitude one.
- Minimizing the integral over the frequency/amplitude curve. With this, instead of including only the maximum values, the whole response is considered.

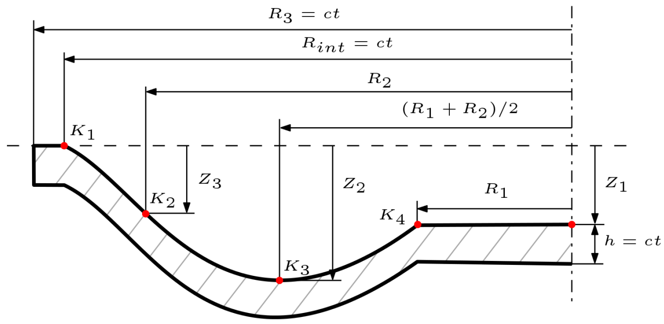

Fig. 8 Curved geometry

After analyzing the behaviour of each formulation, it could be concluded that the first two strategies are too simplistic and have convergence problems. On the other hand, the last two behave satisfactorily, but do not converge to the same designs and the time needed for it varies. This shows that the problem has numerous local minima, and it converges to different ones depending on the cost function and the initial guess used. This also proves that deterministic algorithms, such as SQP, may get trapped in local minima.

Once the most effective cost functions have been determined, the optimization is extended to consider five design variables, as shown in Fig.7. Nevertheless, identifying the frequency peak is not possible in this model due to the mesh changing with the radii in every iteration of the optimization process. This makes it complicated to track the nodes to be used for the MAC. Thus, the last formulation of the above list is used, that is, the integral over the harmonic response curve.

In addition to this, it is observed that the initial guess has an enormous influence in the obtained results. Therefore, it is essential to find the best possible starting point for the optimization process. To do so, the genetic algorithm is used to find possible initial guesses making use of random variations [8]. From this, the best nine solutions are taken and the optimization is performed for each of them, giving the harmonic responses shown in Fig.9.

In order to make sure that the obtained results are valid, the mesh size of the final designs is reduced to check if the response vary in each case. Since the results do not appear to change much, the best result from Fig.9 is chosen as the optimized one, which is Design 9.

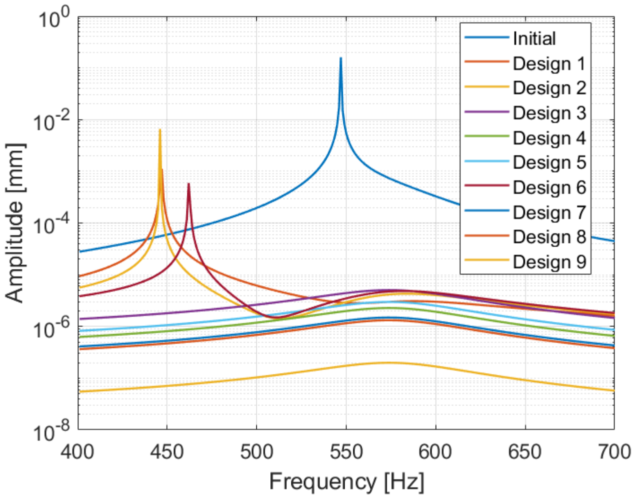

Fig. 9 Harmonic response with the best eight initial guesses obtained from the genetic algorithm

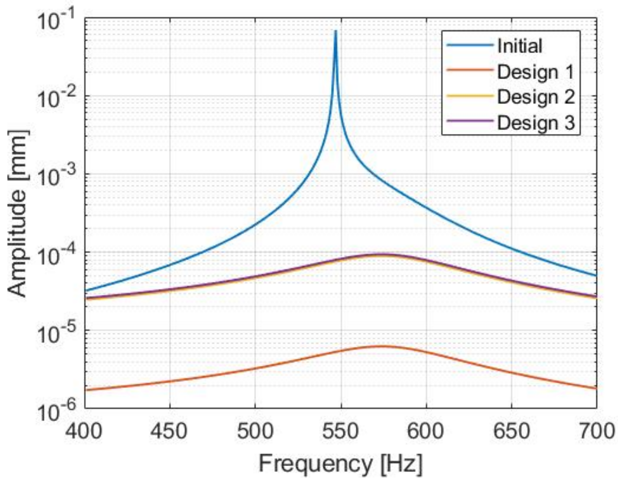

Fig. 10 Harmonic response of the best designs obtained for the curved geometry

The values of the design variables for the optimized result are (h_1/h_2/h_3/R_1/R_2): 6 / 5.8 / 3.7 / 131 / 192 mm and the maximum amplitude value is reduced from $6{\cdot}10^{-2}$ mm to $2.7{\cdot}10^{-7}$ mm.

4.2 Curved cross-section optimization

For this strategy, the insight obtained from the previous approach is used. Before starting the optimization algorithm an initial guess is needed, and thus, genetic algorithm is employed to obtain qualified initial designs.

After obtaining the initial guesses and running the optimization algorithm, the improved designs are obtained. As for the previous strategy, the harmonic response of the best three are shown in Fig. 10.

As it can be observed in Fig. 10, the response with the smallest amplitude is Design 1. The design variables corresponding to this are (Z_1/Z_2/Z_3/R_1/R_2): 0.9 / 2.5 / -0.9 / 66.1 / 128.2 mm. Furthermore, the maximum amplitude value is reduced from $6{\cdot}10^{-2}$ mm to $6{\cdot}10^{-6}$ mm.

4.3 Comparison of optimization strategies

Once the best optimized design is chosen for each strategy, these are compared with respect to the original one. The comparison is divided in two factors: the harmonic response in the range of interest (500-700 Hz); and the Root Mean Squared (RMS) velocity v_{RMS} at 672 Hz. The latter one is used to measure the normal velocity at the top plate, and thus, the radiated sound power which can be obtained from it [10].

For the harmonic response comparison the range of frequencies is increased to have a bigger picture of the behaviour of each design, as shown in Fig.11.

As it can be seen in Fig.11, the stepped cross-section strategy reduces the amplitude response more effectively. In addition, the peak is removed from the range of interest shifting it up to 920 Hz. On the other hand, the curved cross-section does not manage to get such an improvement, moving the pick up to 710 Hz.

To analyze the improvement of the noise emission of the top plate at 672 Hz, the RMS velocity is computed for each design. With this, a comparison of the optimized velocity with respect to the original one is done, as shown in Tab.II.

According to Tab.II, it can be said that the improvement of v_{RMS} is much noticeable with the stepped cross-section.

From this comparison, it can be concluded that the best strategy is the stepped cross-section which has a better performance in both the conducted studies. Therefore, it is decided to manufacture a prototype with this theoretical best design.

5. Prototype

The prototype is manufactured with the final design to validate the optimization suggested to Nilfisk. To do so, a comparison between the expected FE results, initial experimental results and the experimental results of the prototype is performed.

As it can be seen in Fig.12, the results of the are not as expected. The experiments performed with the

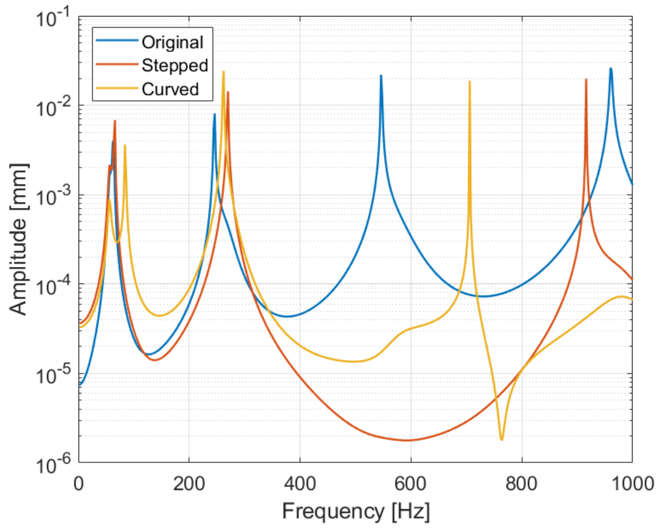

Fig. 11 Harmonic responses of the initial and optimized designs

optimized geometry do not match sufficiently with the expected results. Although the amplitude is lowered in the range of 550-800 Hz, a peak appears at around 520 Hz, where the original design displays a lower amplitude.

There are two significant reasons which could have led to these unexpected results. Firstly, the manufacturing of the optimized model lacks accuracy when compared to the initial model. Due to the high frequency under study, the response is very sensitive to small geometric imperfections [4]. This could cause the transmission of waves to be altered, and thus, affect the results. Secondly, even though the boundary conditions were set as similar as possible, due to the addition of material, the weight of the specimen was increased. This leads to a higher pretension in the springs holding the specimen, which may affect the displacement response.

Besides this, it is important to note that a fixed displacement for the bottom plate is used as boundary conditions for the optimization process. This is preferred instead of the springs because it is computationally more efficient and it did not show to have a significant variation in the results (as discussed in [1]). However, it is necessary to investigate the effect of boundary conditions as a possible source of unexpected FE results.

5.1 Boundary conditions of the FE model

In order to study this, an analysis with the theoretically optimum design is performed with the two different boundary conditions mentioned in Section 2: a fixed displacement, which is used in the optimization; and using soft springs, which is closer to the experimental setup. The latter analysis is shown in Fig.13.

Tab. II Root Mean Square velocities of the initial and optimized designs

Design	v_{RMS} (m/s)	$v_{initial}/v_{optimized}$
Initial	$1.1 \cdot 10^{-4}$	-
Stepped cross-section	$5.2 \cdot 10^{-8}$	2115
Curved cross-section	$3.5 \cdot 10^{-6}$	29

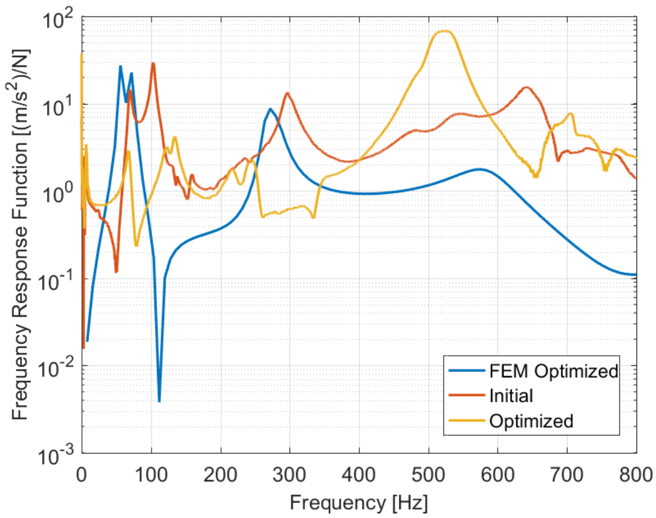

Fig. 12 Comparison

As it can be seen by comparing Fig.13 to Fig.9, the response of Design 9 is not as expected. Additionally, it can be said that the tendencies are similar for other designs (e.g. Design 8), whereas for this particular design it varies considerably. The maximum amplitude of Design 9 shows a jump from $2.7 \cdot 10^{-7}$ mm to $2 \cdot 10^{-4}$ mm, when changing the boundary conditions.

Therefore, it can be concluded that for this specific design the boundary conditions have a big influence in the obtained results. It also showed that the manufactured design may not the optimum one, and thus, a better choice could be made.

5.2 Improved design for the prototype

Instead of the manufactured prototype, Design 8 is proposed, for which optimum values of the design variables are $(h_1/h_2/h_3/R_1/R_2)$: 5.7 / 1 / 4.1 / 122.5 / 124.7 mm. The obtained design in this case shows a slot at around 123 mm, which seems to help to mitigate the propagation of energy waves. Apart from the harmonic response, the RMS velocity of this design is analyzed to make sure that it is indeed lowered. This is shown in Tab.III.

As it can be observed in Tab.III, for the springs

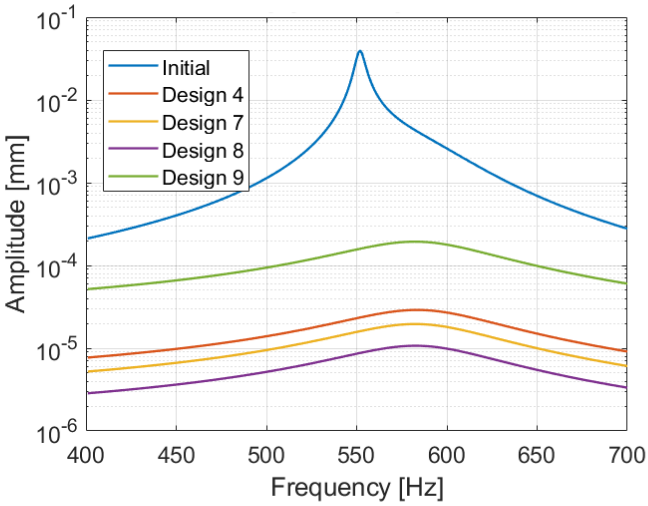

Fig. 13 Springs as boundary conditions

as boundary conditions, the velocity improvement obtained with the new design is clearly better than the manufactured one. However, this needs to be validated and further investigated by experimental work.

6. Conclusions

The main objective of the article was to reduce the vibration levels of a simplified model of an industrial vacuum cleaner. In order to do so, the harmonic response of the system is analyzed, aiming for a reduction in the amplitude by shifting the resonant frequencies of the bottom plate, which is done by reshaping it.

To carry out this strategy, firstly, a FE model of the simplified geometry was created. After validating its performance comparing it to experimental results, an optimization procedure was developed in order to find the best possible design. It was observed that for the case under study, the most effective cost function is the integral under the amplitude response in the range of interest. In addition, due to the procedure being highly dependent on the initial guess, it was essential to find suitable guesses. Once these were found, the optimization was carried out and the best design was chosen.

The new design exhibited a significant reduction in the amplitude response, while shifting the resonant frequencies out of the range of interest. However, after a prototype of this design was manufactured and the validating tests were conducted, this behaviour could not be observed. These unexpected results where attributed to several factors: the boundary conditions used in the FE model might not be realistic enough, the prototype having some manufacturing defects that can alter the response at high frequencies, and the springs in the experimental setup having a higher pretension due to the weight added in the bottom plate.

Tab. III Root Mean Square velocities with springs as boundary conditions

Design	v_{RMS} (m/s)	$v_{initial}/v_{optimized}$
Initial	$7.1 \cdot 10^{-4}$	-
Design 9	$5.9 \cdot 10^{-5}$	12
Design 8	$3.3 \cdot 10^{-6}$	215

Taking this into account, an alternative design was proposed, which showed to behave better than the manufactured one for both boundary conditions. Nevertheless, this has to be validated by experimental work, and thus, check if the optimization strategy works as expected.

Besides this, it is recommendable to improve the FE model including the exact material properties, more accurate damping characteristics (frequency dependant), a more realistic modelling of boundary conditions... in order to have a better correlation between the experimental model and the FE one.

Once the model and the optimization procedure are validated, the considered design variables can be expanded to improve the design even more. This is, having more radial divisions in the stepped cross-section and introducing more key-points in the curved one.

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